Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing

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Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing

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- *Journal of Political Economy (JPE) – Vol. 104, Issue 3 (June 1996).*

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Overview

- Introduction
- The Model
  - The Environment
  - Trading Frictions
  - Equilibrium
  - State Variables
- Simulation Results
  - Representative Agent Baselines
  - Frictionless Trading
  - Trading with Transaction Costs
- Conclusion

Introduction

- A *Complete* market is a market with two specification:
  1. Negligible transaction costs, and therefore also perfect information,
  2. There is a price for every asset in every possible state of the world.
- In this market, a complete set of possible bets can be constructed with existing assets without friction.

- In contrast, in an *Incomplete* market, the number of securities is less than the number of states.
- In *Incomplete* markets, optimal risk sharing (perfect consumption smoothing) is not feasible.

This paper aims to:

- show the extent to which market incompleteness affects asset prices.
- show how market incompleteness distorts risk sharing.
- introduce the effects through which transaction costs impact the aforementioned parameters.

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The Economy

- Two (classes of) agents with different stochastic income realizations, Y_t^i.
- Agents receive income from investments in stocks and bonds.
- Agents are affected by both aggregate (labor income and dividend) shocks and idiosyncratic (labor income) shocks.
- Agents are not allowed to write contracts contingent on future labor income.

The Economy - Continued

- Each agent's preference over consumption is:

$$U_t^i \equiv \mathbb{E}\{\sum_{\tau=0}^{\infty} \beta^\tau \frac{(c_{t+\tau}^i)^{1-\gamma} - 1}{1-\gamma} \mid \mathcal{F}(t)\}, \quad \gamma > 0 \tag{1}$$

- Constraints of the economy (for all $t = 0, 1, 2, ...$):

$$\begin{aligned} c_t^i + p_t^s s_{t+1}^i + p_t^b b_{t+1}^i + \kappa(s_{t+1}^i, s_t^i, \mathbf{Z}_t) + \omega(b_{t+1}^i, b_t^i, \mathbf{Z}_t) \\ \leq s_t^i(p_t^s + d_t) + b_t^i + Y_t^i, \end{aligned} \tag{2}$$

$$s_t^i \geq K_t^s, \tag{3}$$

$$b_t^i \geq K_t^b, \tag{4}$$

- Market clearing requires (for all $t = 0, 1, 2, ...$):

$$b_t^1 + b_t^2 = 0, \tag{5}$$

$$s_t^1 + s_t^2 = 1. \tag{6}$$

Trading Frictions

In the base case model:

- Stock market transaction costs:

$$\kappa(s^i_{t+1}, s^i_t, \mathbf{Z}_t) = k_t[(s^i_{t+1} - s^i_t)p^s_t]^2.$$

  Stock market trading cost as a percentage of the value of shares traded is: $k_t|s^i_{t+1} - s^i_t|p^s_t$.

- Bond market transaction costs:

$$\omega(b^i_{t+1}, b^i_t, \mathbf{Z}_t) = \Omega_t \min\{0, b^i_{t+1}p^b_t\}^2.$$

  Bond market trading cost as a percentage of the value of bondss traded is: $\frac{\Omega_t|b^i_{t+1}|p^b_t}{2}$.

Equilibrium Conditions

- For all $t = 0, 1, 2, \ldots$:

$$\sum_{i=1,2} [c_t^i + \kappa(s_{t+1}^i, s_t^i, \mathbf{Z}_t) + \omega(b_{t+1}^i, b_t^i, \mathbf{Z}_t)] = d_t + Y_t^1 + Y_t^2, \quad (7)$$

$$\begin{aligned} \beta\, \mathbb{E}\{u^{'}(c_{t+1}^i)[p_{t+1}^s + d_{t+1} - \kappa_2(s_{t+2}^i, s_{t+1}^i, \mathbf{Z}_{t+1})] \,|\mathcal{F}(t)\} \\ = [p_t^s + \kappa_1(s_{t+1}^i, s_t^i, \mathbf{Z}_t)] u^{'}(c_t^i), \quad & (8) \\ or : s_t^i = K_t^s, \quad & (8') \end{aligned}$$

$$\begin{aligned} \beta\, \mathbb{E}\{u^{'}(c_{t+1}^i)[1 - \omega_2(b_{t+2}^i, b_{t+1}^i, \mathbf{Z}_{t+1})] \,|\mathcal{F}(t)\} \\ = [p_t^b + \omega_1(b_{t+1}^i, b_t^i, \mathbf{Z}_t)] u^{'}(c_t^i), \quad & (9) \\ or : b_t^i = K_t^b. \quad & (9') \end{aligned}$$

- At each t, the unknowns are: $c_t^i, p_t^s, p_t^b, s_t^i, b_t^i$.

State Variables of The Model

The exogenous state variables at time t are defined by:

- Aggregate labor income and aggregate dividend income: Y_t^l, D_t^a,
- Total aggregate income: $Y_t^a \equiv Y_t^l + D_t^a$,
- Growth rate in aggregate income: $\gamma_t^a \equiv Y_t^a / Y_{t-1}^a$,
- Dividend's share in aggregate income: $\delta_t \equiv D_t^a / Y_t^a$,
- Individual i's labor income as a fraction of aggregate labor income: $\eta_t^i \equiv Y_t^i / Y_t^a$.

* The state of the economy at time t is given by $[\gamma_t^a, \delta_t, \eta_t^i]'$, estimated using the PSID dataset in 1969-84 and the NIPA dataset in 1947-92.

Calibration of the states is done:

- In a Markov Chain with 8 states estimated by a VAR.
- Using NIPA dataset in 1947-92.
- In base case model and a cyclical distribution case (CDC) model in which imposed individuals' shocks are heteroskedastic.

Overview

Representative Agent Baselines

- In the baseline model with no frictions (complete markets):

MOMENTS IMPLIED BY THE COMPLETE MARKETS CASE

		Aggregate		Individual	
Moment	Data (1)	Base Case (2)	Cyclical Distribution Case (3)	Base Case (4)	Cyclical Distribution Case (5)
Consumption growth:					
Average	.020	.018	.015	.018	.015
Standard deviation	.030	.028	.028	.217	.259
Bond return:					
Average	.008	.080	.077	.055	.041
Standard deviation	.026	.009	.012	.175	.213
Stock return:					
Average	.089	.082	.078	.137	.152
Standard deviation	.173	.029	.028	.375	.441

Figure 1: Moments of the data versus moments of the complete markets case

- The model explains only the first moment of the data and not the second.

Frictionless Trading

- In the frictionless trading model:

MOMENTS IMPLIED BY THE FRICTIONLESS MODEL

Moment	Base Case	Cyclical Distribution Case
Consumption growth:		
Average	.018	.016
Standard deviation	.044	.045
Bond return:		
Average	.077	.073
Standard deviation	.012	.017
Stock return:		
Average	.079	.073
Standard deviation	.032	.030
Bond trades (percentage of consumption):		
Average	.045	.042
Standard deviation	.060	.052
Stock trades (percentage of consumption):		
Average	.131	.146
Standard deviation	.066	.082

Figure 2: Simulated moments of the frictionless trading model

- Introduction of transaction costs in only one of the markets has negligible effects in asset prices.

Transaction Costs in Both Markets

- The *"direct"* effect of transaction costs:
  - Agents equate net-of-cost margins in markets. Therefore, with higher cost in the markets, the risk free rate decreases.

$$r^s - \kappa = r^b : \quad \kappa \uparrow \Rightarrow r^b \downarrow .$$

  Where r^s, and r^b are stock market and bond market returns, respectively. Note that $r^b =$ lending rate which is the return of lending bonds!

- This phenomena is known as the risk-free-rate puzzle.

Transaction Costs in Both Markets

- The direct effect can be seen in the base case simulations:

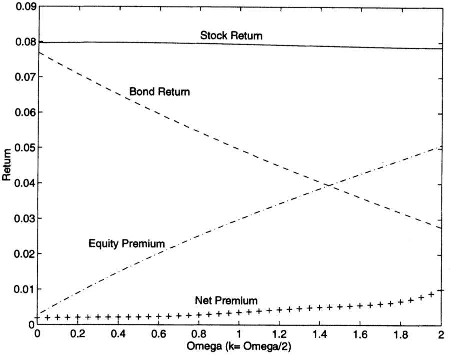

Figure 3: Base case, returns

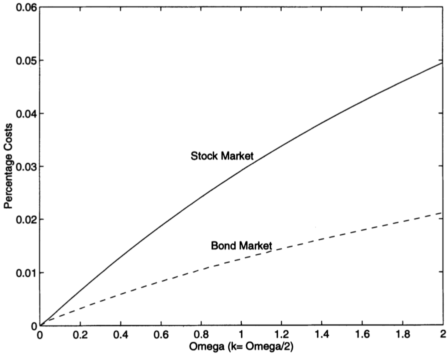

Figure 4: Base case, trading costs

Transaction Costs in Both Markets

- The direct effect can be seen in the CDC simulations:

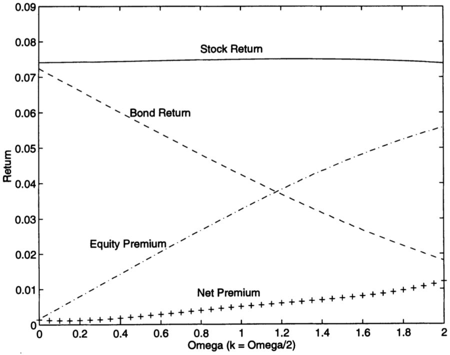

Figure 5: CDC, returns

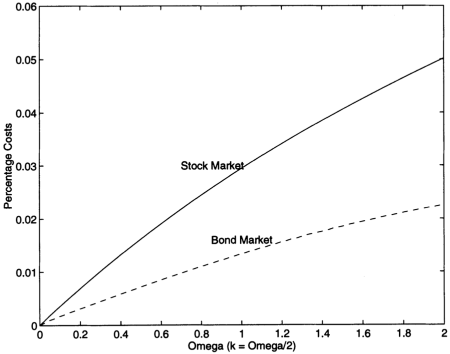

Figure 6: CDC, trading costs

Transaction Costs in Both Markets

- Following an increase in transaction costs, average trading (as a percentage of total income) decreases while consumption growth volatility increases.

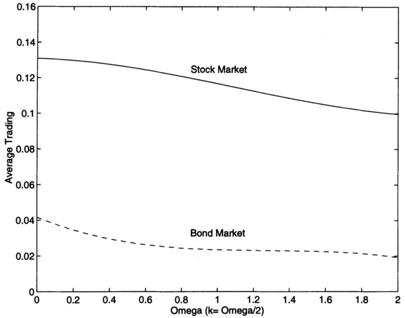

Figure 7: Base case, average trading

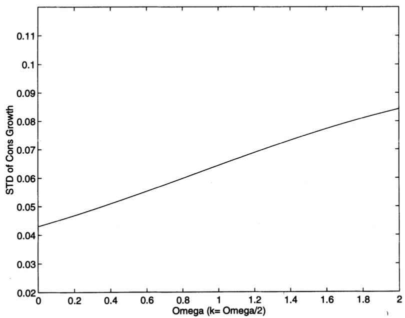

Figure 8: Base case, STD of cons. growth

Transaction Costs in Both Markets

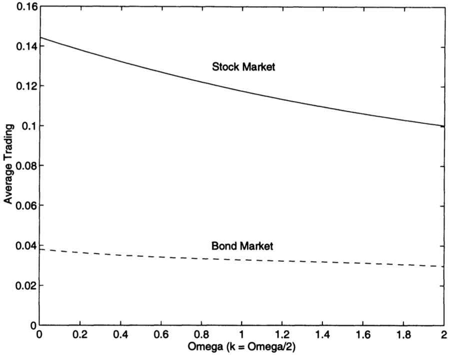

Figure 9: CDC, average trading

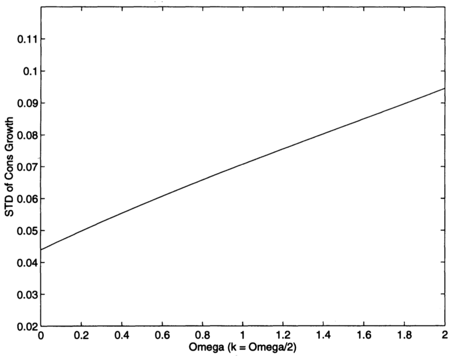

Figure 10: CDC, STD of cons. growth

Transaction Costs in Both Markets

- The *"indirect"* effect of transaction costs:
  - Agents require more premium for more volatile consumption due to higher transaction costs. This is reflected in the *net* premium.
- Define:

$$r_{t,t+1}^{s,net} \equiv \frac{p_{t+1}^s + d_t t + 1 + 2k_{t+1}(s_{t+2}^i - s_{t+1}^i)(p_{t+1}^s)^2}{p_t^s + 2k_t(s_{t+1}^i - s_t^i)(p_t^s)^2} - 1, \qquad (10)$$

$$r_{t,t+1}^{b,net} \equiv \begin{cases} \frac{1}{p_t^b} & b_{t+1}^i \geq 0 \\ \frac{1}{p_t^b + s\Omega_t b_{t+1}^i (p_t^b)^2} & b_{t+1} < 0. \end{cases} \qquad (11)$$

Note that (10) and (11) satisfy (8) and (9), respectively.

- The indirect effect is roughly 20% of the equity premium in both cases.

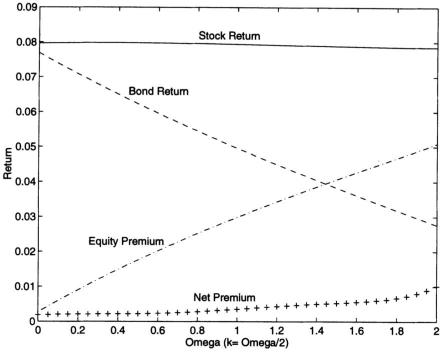

Figure 11: Base case, returns

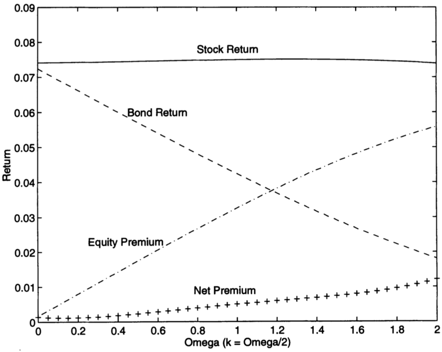

Figure 12: CDC, returns

Robustness to cost specifications

- Now, assuming the cost function is quadratic for small transactions and becomes linear afterwards:

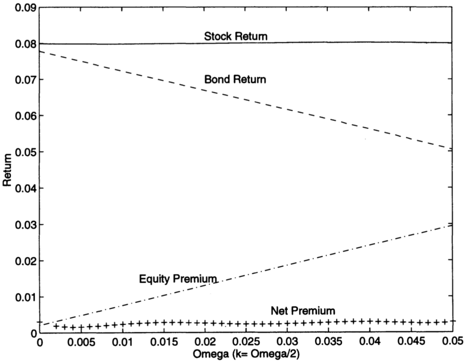

Figure 13: Linear costs, base case, returns

Transaction Costs in Both Markets and Symmetric Bond Market Costs

- Now, suppose both the lender and the borrower bear the cost of transaction in bond market:

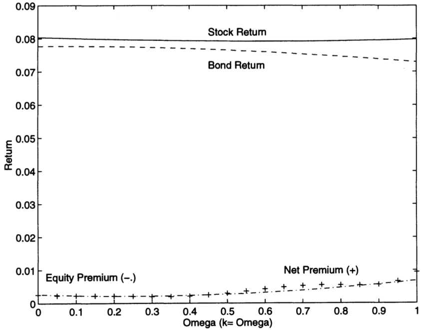

Figure 14: Base case, symmetric bond costs, returns

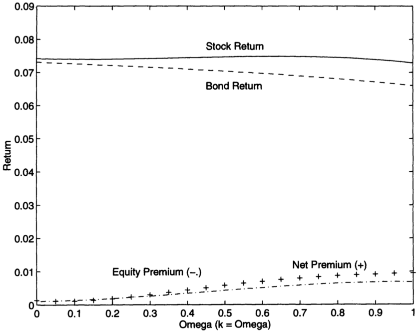

Figure 15: CDC, symmetric bond costs, returns

Transaction Costs in Stocks and No Borrowing

- This is the extreme case of when no borrowing is allowed.
- For any level of costs, the level of (shadow) bond return and equity premium is larger than the case with borrowing.

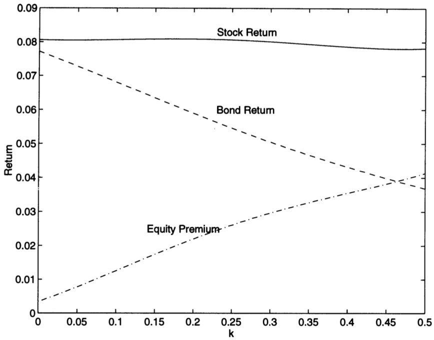

Figure 16: Base case, no bonds, returns

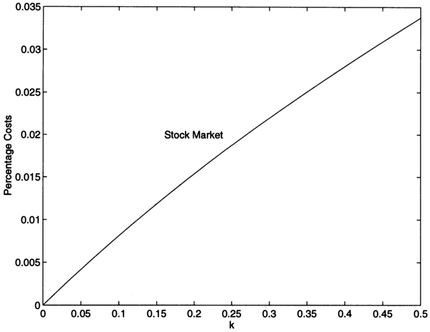

Figure 17: Base case, no bonds, average costs

The Effect of an Outside Supply of Bonds

- With a variable change and assuming that the transaction cost is paid as a lump-sum tax to the government, the results will be the same as when no borrowing is allowed.

Overview

Conclusion

In this paper:

- The effect of incompleteness of market on asset prices and risk sharing in presence of both aggregate and idiosyncratic shocks has been studied.
- "direct" effect of costs on equity premium has been discussed:
  - Due to this effect, the market with lower cost of transaction will have lower market rate of return.
  - The size of this effect varies with the structure of the costs.
- "indirect" effect of costs on equity premium has been pointed out:
  - Following an increase in the transaction cost, covariance of consumption and returns increases.
  - This indirect increase in the systematic component of (stock) returns, explains about 20% of the equity premium.
  - It is not sensitive to the cost structure.

Thanks for your attention!