- WHY do we use OLG ?
  + agents don't live infinitely (they die).
  + new agents are born over time.
  + considers the dynamic interaction between generations [olders' decisions affect youngers]
  + tractable alternative to infinitely lived representative agent.
  + Break First Welfare Theorem.
  + Break Ricardian equivalence and the implication that $k^* < k^g$
  + Real world implications : existence of rational bubbles & pension schemes.

    ↓

    without any behavioral assumptions on preferences.

  * The number of generations "N" is determined by "N" different types of agents (or behaviors) that one wants to show.

- The Environment
  + people live for N = 2 periods (old and young). Additively seperable utility:

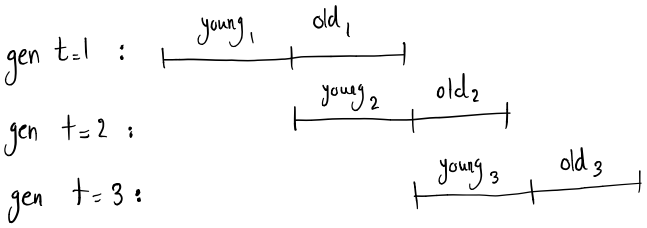
    agents of gen. t : $U(c_t^y, c_{t+1}^o) = u(c_t^y) + u(c_{t+1}^o)$

  + No "Altruism" : no bequests.
  + for simplicity assume zero population growth. (N young , N old)
  + pure exchange economy : abstract from production function.

- Conclusion
  + models can be solved for stationary / non-stationary equilibria.

– First, remember: 1st welfare theorem: C.E. $\Longrightarrow$ P.O. (lcl non saturation)

2nd " : P.O. $\Longrightarrow$ C.E. (concavity of u)



A competetive eq. is allocation $\{c_t^y, c_{t+1}^o, a_{t+1}\}_{t=0}^{\infty}$ and prices $\{r_{t+1}\}_{t=0}^{\infty}$ s.t. :

(I) Max $u(c_t^y) + \beta u(c_{t+1}^o)$

s.t. $c_t^y + a_{t+1} = e_t^y - T_t^y$ $[\lambda]$

$c_{t+1}^o = e_{t+1}^o + (1+r_{t+1})a_{t+1} - T_{t+1}^o$ $[\mu]$

$$\Rightarrow \left.\begin{array}{l} u'(c_t^y) = \lambda \\ u'(c_{t+1}^o) = \mu \\ -\lambda + (1+r_{t+1})\mu \end{array}\right\}$$

$$\Rightarrow \quad EE: \frac{u'(c_t^y)}{u'(c_{t+1}^o)} = (1+r_{t+1}) = -\frac{c_{t+1}^o - e_{t+1}^o}{c_t^y - e_t^y} = -\frac{z_{t+1}}{z_t}$$

$$\Rightarrow \quad u'(z_t + e_t^y)\, z_t + u'(z_{t+1} + e_{t+1}^o)\, z_{t+1} = 0 \qquad (*)$$

(II) $0 = \sum_{i=1}^{N} a_{t+1} \Rightarrow a_{t+1} = 0 \quad \forall t$, $T_t^y + T_t^o = 0 \overset{\text{sup.}}{\Longrightarrow} T_t^y = T = -T_t^o \quad \forall t$

$N c_t^y + N c_t^o = N e_t^y + N e_t^o$

– now, sup. $u(c) = \ln c$, $w_t^o = w^o < w^y = w_t^y$:

$[a_{t+1} = 0] \Rightarrow \frac{w^o}{w^y} = 1 + r_{t+1} = 1 + r < 1 \iff r < 0$, propose a reallocation

no-trade economy

$\Longrightarrow$ define $\Delta = \frac{w^y - w^o}{2} \Rightarrow \begin{array}{l} \tilde{c}^y = w^y - \Delta \\ \tilde{c}^o = w^o + \Delta \end{array} \Rightarrow u(\tilde{c}^y) + u(\tilde{c}^o) = 2u\left(\frac{w^y + w^o}{2}\right)$

$> u(w^y) + u(w^o)$

$\Longrightarrow$ C.E. is not pareto optimal. (works for $\Delta^* \in (0, \Delta)$)

this is called "Dynamic Inefficiency" $\longrightarrow$ Due to the missing market

- So far, I calculated stationary equilibria, what about non stationary eq.?

$$z_t + \mathcal{I}_t = 0 \implies z_t = -\mathcal{I}_t$$

$$\xrightarrow{*} u'(z_t + e_t^y) z_t - u'(-z_{t+1} + e_{t+1}^o) z_{t+1} = 0 \implies z_{t+1} = G(z_t)$$

[if $u'(\cdot)$ is invertible, ...]

the sequence $\{z_t\}_{t=1}^{\infty}$ gives the non-stationary eq.

Applications $\longrightarrow$ Learning true parameter of s.t. over time by different generations.

+ Now, introduce government, but not yet debt:

$$EE: \quad \frac{w^o + T}{w^y - T} = 1 + r \qquad \frac{\partial r}{\partial T} > 0$$

- again introducing taxes might be pareto improving if we have dynamic inefficiency.

+ Now, introduce government debt, price in $t = t_0$ is q_{t_0} and claims one unit of cons. in $t = t_0 + 1$:

$$1 = (1 + r_{t+1}) q_t$$ [no arbitrage implies return on debt = return on private lending]

$$\implies \frac{1}{q_t} = 1 + r_{t+1}$$

- government's budget constraint : $b_t q_t + T_t^y + T_t^o = b_{t-1}$ [mixed financing]

old " " : $c_t^o = w_t^o - T_t^o + b_{t-1}$

young " " : $c_t^y = w_t^y - T_t^y - q_t b_t$ [only the young buy the bond]

- market clearing for bonds : supply of bonds : $q_t b_t = b_{t-1} - T_t^y - T_t^o$
demand for bonds : $S_t = c_t^y - w_t^y + T_t^y$

$\Rightarrow$ [market clearing condition] : $c_t^y = w_t^y + T_t^o - b_{t-1}$

- in eq. : $$\frac{1}{q_t} = 1 + r_{t+1} = \frac{u'(c_t^y)}{u'(c_{t+1}^o)}$$

$$c_t^o = w_t^o - T_t^o + b_{t-1}$$

$$c_t^y = w_t^y - T_t^y - b_{t-1}$$

Note that T & b_{t-1} can not be too large.

- Now, supp. $T = 0$: if $r_{t+k} = 0 \rightarrow$ debt would be constant overtime $= b_0$

if $r_{t+k} > 0 \rightarrow$ X

if $r_{t+k} < 0 \rightarrow$ ✓ debt converges to zero

* Ricardian equivalence breaks down.

+ Now, let's suppose in contrast to bonds, we have a long lived asset A (a house, land, ...).
  - A has price $P_{t+1}^{e,i} = P_{t+1}^e$ in period $t+1$ and pays dividend $d_{t+1} = d$
  - due to no arbitrage return on A is equal to private lending return : r_{t+1}
  - Assume perfect foresight : $P_{t+1}^e = P_{t+1}$

- individual budget constraint : $c_t^y = w_t^y - P_t a_{t+1}$
$$c_{t+1}^o = w_{t+1}^o + (P_{t+1} + d) a_{t+1}$$

- Market clearing : $a_{t+1} = A$ [agg. demand of asset = agg. supply]

interest rate is given by : $$\frac{u'(c_t^y)}{u'(c_{t+1}^o)} = 1 + r_{t+1}$$

the price sequence satisfies : $$P_t = \frac{P_{t+1} + d}{1 + r_{t+1}}$$

- let us only investigate stationary eq^h:

$$p = \frac{p+d}{1+r} \quad , \quad c^y = w^y - pA \quad , \quad c^o = w^o + (p+d)A$$

$$\frac{u'(w^y - pA)}{u'(w^o + (p+d)A)} = 1+r = 1 + \frac{d}{r} \implies p = \frac{d}{r}$$ price is discounted future dividends.

- suppose $d=0$, $u(c) = \ln c$: $\frac{w^y - pA}{w^o + pA} = 1 \implies w^y - pA = w^o + pA \implies p = \frac{w^y - w^o}{2A} \neq 0$!

* This is known as "Rational Bubble" where $d=0$ and $p \neq 0$.

  Note that consumption is equal in each period $\implies$ eff., not necessarily a bad thing.

- Generally, rational bubble can rise whenever $r<g$.

- bursting happens when people start to think $p=0$.

Applications: Macro $\longrightarrow$ public finance [Diamond 1965]

the role of money [Samuelson 1958] – Monetary economics

pension schemes

growth (demographic transition, capital accumulation, ...)

Consumption smoothing & life cycle theories

(sunspot equilibria)

Finance $\longrightarrow$ liquidity & bubbles

market incompleteness & effect on hedging

Check with Ramtin later for the controversy ...