Efficient Matching Under Distributional Constraints: Theory And Applications

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Overview

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Motivation

- Many real matching markets are subject to distributional constraints.
- Examples: medical residency, state-financed seats in college admissions, military service, etc.
- Prior mechanisms in practice and the literature yield avoidable inefficiency and instability under distributional constraints.

This Paper

- Studies matching under distributional constraints using Japan Residency Matching Program (JRMP) as a baseline.
- Shows how conventional mechanisms fail under these circumstances.
- Introduces a new mechanism that has the desirable properties of efficiency, stability, and strategy-proofness.
- Shows that some results in the standard environment fail to hold under constraints.

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Literature Review

- The main results of this paper contribute to the literature of matching with contracts. Mainly, this work is closely related to Hatfield, Milgrom (2005).
- Milgrom (2009) and Budish, Che, Kojima, and Milgrom (2013) consider object allocation mechanisms with restrictions. However, stability is not studied in those papers.
- Abraham , Irving, Manlove (2007) and Sonmez, Unver (2006) study a similar concept in matchings. However, the former has identical preferences and the latter uses a different concept of stability.

Overview

Notations

- A finite set R of different regions.
- There exists a regional cap q_r for each region $r \in R$.
- A finite set D of doctors each with strict preferences $\forall d \in D : \succ_d$.
- A finite set H of hospitals. Each hospital h has a '*physical*' capacity of q_h with responsive preferences $\forall h \in H : \succ_h$.
- In JRMP, each hospital has an exogenously given '*target*' capacity $\bar{q}_h \leq q_h$.
- Let H_r denotes the set of hospitals in region $r \in R$, and $r(h)$ denotes the region r such that $h \in H_r$.

Definitions

Definition 1

A matching μ is **feasible** if $|\mu_r| \leq q_r$ for all $r \in R$, where $\mu_r = \cup_{h \in H_r} \mu_h$.

Definition 2

A feasible matching μ is **(constrained) efficient** if there is no feasible matching μ' such that $\mu'_i \succeq_i \mu_i$ for all $i \in D \cup H$ and $\mu'_i \succ_i \mu_i$ for some $i \in D \cup H$.

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The JRMP Mechanism

- The Japan Residency Matching Program (JRMP) mechanism is a rule that produces the matching resulting from the deferred acceptance algorithm except that, for each $h \in H$, it uses $\bar{q}_h$ instead of q_h as the hospital's capacity.
- The JRMP mechanism does not necessarily produce an efficient matching.

Inefficiency of The JRMP Mechanism

Example 1

There is one region r with regional cap $q_r = 10$, in which two hospitals h_1 and h_2 reside. Suppose there are 10 doctors. The agents' preference profile $\succ$ is as follows:

$$\succ_{d_j}: h_1 \quad \text{if } j \leq 3, \qquad \succ_{h_1}: d_1, \ldots, d_{10}, \quad q_{h_1} = 10, \bar{q}_{h_1} = 5,$$
$$\succ_{d_j}: h_2 \quad \text{if } j \geq 4, \qquad \succ_{h_2}: d_1, \ldots, d_{10}, \quad q_{h_2} = 10, \bar{q}_{h_2} = 5.$$

The resulting matching using the JRMP mechanism is:

$$\mu = \begin{pmatrix} h_1 & h_2 & \emptyset \\ d_1, d_2, d_3 & d_4, d_5, d_6, d_7, d_8 & d_9, d_{10} \end{pmatrix}.$$

However, the matching μ' is feasible and every agent is weakly better:

$$\mu' = \begin{pmatrix} h_1 & h_2 \\ d_1, d_2, d_3 & d_4, d_5, d_6, d_7, d_8, d_9, d_{10} \end{pmatrix}.$$

Overview

Goal Setting: Stability and Strategy-Proofness

- This paper aims to introduce a new mechanism that satisfies both strategy-proofness and a notion of stability.
- Kamada, Kojima (2017): Stability concept can be defined in various ways and to different extents.

Efficiency versus Stability

Definition 3

A matching μ is **stable** if it is feasible, individually rational, and if there exists a blocking pair (d, h), then:

(i) $d' \succ_h d$ for all doctors $d' \in \mu_h$,

(ii) $|\mu_{r(h)}| = q_{r(h)}$.

In words, the blocking pair would not change the matches of other doctors and hospitals and the formation of the block would violate the regional cap.

Theorem 1

Any stable matching is (constrained) efficient.

The Flexible Deferred Acceptance

- The paper introduces a mechanism that is consistent with the new stability concept, namely, the *Flexible Deferred Acceptance* (FDA) mechanism.
- The Flexible Deferred Acceptance mechanism is the mechanism that produces for each input, the matching that terminates the following algorithm:

The Flexible Deferred Acceptance Algorithm

- The FDA algorithm:
  1. Begin with an empty matching μ such that $\mu_d = \emptyset$ for all $d \in D$.
  2. Choose a doctor who is currently not tentatively matched to any hospital and who has not applied to all acceptable hospitals yet. If such a doctor does not exist, the algorithm is terminated.
  3. let d apply to the most preferred hospital $\bar{h}$ at $\succ_d$ among the hospitals that have not rejected her so far. Let r be the region such that $\bar{h} \in H_r$.
  4. For each $h \in H_r$, letting D'_h be the set of doctors who have applied to but have not been rejected by h so far and are acceptable to h, choose $D_h^{''}$ such that: $D_h^{''} \subset D'_h, |D_h^{''}| = min\{\bar{q}_h, |D'_h|\}$, and $d \succ_h d'$ for any $d \in D_h^{''}$ and $d' \in D'_h \setminus D_h^{''}$.
  5. Start with a tentative match $D_h^{''}$ for each hospital $h \in H_r$. Hospitals take turns to choose (one doctor at a time) the best remaining doctor in their current applicant pool. Repeat the procedure until the regional quota q_r is filled or the capacity of the hospital is filled or no doctor remains to be matched. All other applicants are rejected.

Main Results

- The flexible deferred acceptance algorithm stops in finite steps.

Theorem 2

The flexible deferred acceptance mechanism produces a **stable** and **efficient** matching for any input and is **group strategy-proof for doctors**.

Theorem 3

For any preference profile $\succ$:
For each doctor $d \in D$: $\phi^{DA}(\succ)(d) \succeq_d \phi^{FDA}(\succ)(d) \succeq_d \phi^{JRMP}(\succ)(d)$.

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Characteristics of The FDA Mechanism

- The FDA mechanism has some characteristics.

Proposition 1

If $|\mu_h^{FDA}| < \bar{q}_h$, then for any $d \in D$ such that $\mu_d^{FDA} \neq h$, either d is unacceptable to h or d prefers its current match to h.

Proposition 2

1. If $|\mu_h^{FDA}| < \bar{q}_h$ for $h \in H$, then $\mu_h^{DA} \subseteq \mu_h^{FDA}$.
2. If $|\mu_r^{FDA}| < q_r$ for $r \in R$, then $\forall h \in H_r : \mu_h^{FDA} \succeq_h \mu_h^{DA}$. Moreover, $|\mu_h^{FDA}| \geq |\mu_h^{DA}|$.

Characteristics of The Environment

- Generally, matching in presence of constrained quotas has the following implications:
  - The conclusion of the Rural Hospital Theorem does not hold in this environment.
  - There does not necessarily exist a doctor-optimal stable matching. Neither does there exists a hospital-optimal stable matching.
- These, and similar arguments, tell us that there is room for future research in this area!

The Rural Hospital Theorem Results Does NOT Hold

Example 2

There is 1 region with regional cap $q_r = 1$, in which two hospitals h_1 and h_2, with capacities of $q_{h_1} = q_{h_2} = 1$ reside. Let the target capacities be arbitrary. Assume the following preferences:

$$\succ_{d_1}: h_1, \qquad \succ_{h_1}: d_1,$$
$$\succ_{d_2}: h_2, \qquad \succ_{h_2}: d_2.$$

There could be two stable matchings:

$$\mu = \begin{pmatrix} h_1 & h_2 & \emptyset \\ d_1 & \emptyset & d_2 \end{pmatrix}, \quad \mu' = \begin{pmatrix} h_1 & h_2 & \emptyset \\ \emptyset & d_2 & d_1 \end{pmatrix}.$$

Therefore, the results of the Rural Hospital Theorem does not hold.

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Conclusion

- We studied matching under distributional constraints.
- We introduced a new notion of stability.
- We discussed that the outcome of FDA mechanism can outperform the outcome of both DA and JRMP mechanisms in a sense of stability that respects the regional caps.
- We pointed that, in some situations, the outcome of FDA mechanism can even be weakly better than DA mechanism for the hospitals.
- We showed that, some general conclusions of matching in the standard environment do not hold in matching with distributional constraints. This, opens doors of future research in this literature.

Thanks for your attention!