Contract Theory - Moral Hazard Presentation

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Papers

First Paper:

- **“Efficient and Nearly Efficient Partnerships”** *by: Legros, Matthews*
- *Published in:* **The Review of Economic Studies (1993)**

Second Paper:

- **“A Comparison of Tournaments and Contracts”** *by: Green, Stokey*
- *Published in:* **Journal of Political Economy (1983)**

Overview

Introduction

- If the partners' actions are not verifiable, partnerships are inefficient due to deviation by players. Therefore, we aim to design a contract such that nobody deviates in the equilibrium.

This paper:

- Studies deterministic partnerships in which risk neutral partners jointly produce according to a nonstochastic technology and share the resulting output.
- Gives conditions under which efficiency is attained.

Notation and Assumptions

- $N = \{1, 2, ..., n\}$ risk neutral partners ($n \geq 2$).
- Set of possible actions for each partner is A_i.
- Disutility function for each partner $v_i : A_i \to \mathbb{R}$.
- Production function $f : \times_{i \in N} A_i \to \mathbb{R}, y = f(a)$.

- *Efficient actions* are those which maximize:

$$W(a) \equiv y(a) - \Sigma v_i(a_i).$$

  Efficient actions are supposed to exist and are unique.
- A *sharing rule*, $s : f(A) \to \mathbb{R}^n$, determines each partner's share of the output: $s_i(y)$ and satisfies the budget constraint: $\forall y : \Sigma s_i(y) = y$.

Notation and Assumptions - Continued

- To each sharing rule s corresponds a *partnership game*, $\Gamma(s)$.
- The set of strategies for each partner is A_i, and his payoff function is:

$$u_i(a) \equiv s_i(f(a)) - v_i(a_i)$$

* *Efficiency* is *sustainable* if a sharing rule s exists such that a^* is a Nash equilibrium of $\Gamma(s)$.
* *Approximate Efficiency* is *sustainable* if $\forall \epsilon > 0$, a sharing rule s exists such that $\Gamma(s)$ has a mixed strategy equilibrium, $P = (P_1, P_2, ..., P_n)$, satisfies:

$$\mathbb{E}_P \, W(\tilde{a}) > W(a^*) - \epsilon,$$

where $\tilde{a}$ is a random variable with distribution P.

Example 1 - Leontief Production Function

- Let $A_i = \mathbb{R}_+, f(a) = min(a_1/\theta_1, ..., a_n/\theta_n)$.
- Disutility function is strictly convex, differentiable, and satisfies $v_i'(0) = 0$ for all i.
- Efficient actions are: $a_i^* = \theta_i y^*$ for all i.
- Output is determined by $\Sigma \theta_i v_i'(\theta_i y^*) = 1$.
- Define $s_i(y) \equiv \theta_i v_i'(\theta_i y^*) y$.
  Thus, we can write have:

$$u_i(a_i, a_{-i}^*) = \theta_i v_i'(\theta_i y^*) \times min(a_i/\theta_i, y^*) - v_i(a_i).$$

- Best reply of partner i is $a_i = \theta_i y^* = a_i^*$, and efficiency is sustained.

Example 2 - Increasing Production Function and Compact Action Set

- Let $n = 2, A_i = [0, 2], f(a) = a_1 + a_2$, and $v_i(a_i) = a_i^2/2$. Then $a^* = (1, 1)$ and it's not sustainable.
- The following mixed strategy is sustainable as an equilibrium:

$$P_1(0) = P_1(2) = \delta, P(1) = 1 - 2\delta, \ and \ P_2(1) = 1$$

  Note that this strategies converge to a^* as $\delta \to 0$.
- Define the sharing rule as:

$$s_1(y) = (y-1)^2/2 \ and \ s_2(y) = y - s_1(y) \quad for \ y \in [1, 3],$$

$$s_1(y) = y + F \ and \ s_2(y) = -F \quad for \ y \notin [1, 3].$$

  Then, for large enough F, P_1 is a best reply to P_2 and vice versa.

Further Notation

- Define the set of outputs that partner i can achieve by a unilateral deviation from a^*:

$$Y_i \equiv \{y \in \mathbb{R} \,|\, f(a_i, a^*_{-i}) \; for \; some \; a_i \in A_i\}.$$

  And define $Y \equiv \cap_i Y_i$. Note that $y^* \in Y$.

- Define:

$$c_i(y) \equiv inf\{v_i(a_i) | f(a_i, a^*_{-i}) = y, a_i \in A_i\}.$$

  Note that $c_i(y^*) = v_i(a_i^*)$.

- If outputs were shared equally, the most partner i could gain by a unilateral deviation from a^* that gives output y would be:

$$g_i(y) \equiv [y/n - c_i(y)] - [y^*/n - v_i(a_i^*)]$$

  Therefore, the *average gain* from deviating to y is:

$$g(y) \equiv \frac{1}{n}\Sigma g_i(y) = \frac{1}{n}[y - \Sigma c_i(y) - W(a^*)].$$

Main Results

* *Theorem 1*: Efficiency is sustainable if and only if:

$$g(y) \leq 0 \ \forall y \in Y \ \textit{satisfying} \ y < y^*.$$

Main Results - Continued

* *Theorem 2*: Approximate efficiency is sustainable if:
$A1 : A_i \subset \mathbb{R} \ \forall i \in N,$
$A2 : f : A \to \mathbb{R} \textit{ is strictly increasing},$
hold, and $\underline{a}_1 = min(A_1)$ and $\bar{a}_1 = max(A_1)$ exist and are finite.
If $a_1^* \in (\underline{a}_1, \bar{a}_1)$ and $\delta \in (0, 1/2)$, a fine $F < \infty$ exists such that the strategies defined by:

$$P_1(\underline{a}_1) = P_1(\bar{a}_1) = \delta,\ P_1(a_1^*) = 1 - 2\delta,\ and\ P_i(a_i^*) = 1 \ \forall i > 1,$$

are an equilibrium for the sharing rule define by:

$$y \in Y_1 \Rightarrow s_1(y) = c_1(y),\ s_i(y) = (y - c_1(y))/(n-1) \ \forall i > 1,$$

$$y \notin Y_1 \Rightarrow s_1(y) = y + (n-1)F,\ s_i(y) = -F \ \forall i > 1.$$

Conclusion

In this paper:

- A necessary and sufficient condition is provided for a partnership to sustain full efficiency.
  - As in the case of Leontief Production Function example.
  - Consistent with Holmstrom (1982)'s Budget Breaker that has no influence on the output, but sets the budget such that everybody implements their first best effort.

- Approximate efficiency is shown to be achievable in large class of partnerships.
  - As in the case of Increasing Production Function and Compact Action Set example.
  - *Limited Liability* may restrict the degree to which we approximate efficiency. Hence, it's good to include some wealthy partners in a partnership!

Implications for Other Literatures

1. Bull, Jesse, and Joel Watson. "Evidence disclosure and verifiability." Journal of Economic Theory 118.1 (2004): 1-31.
2. Desai, Mihir A., C. Fritz Foley, and James R. Hines Jr. "The costs of shared ownership: Evidence from international joint ventures." Journal of Financial Economics 73.2 (2004): 323-374.
3. Legros, Patrick, and Andrew F. Newman. "Wealth effects, distribution, and the theory of organization." Journal of Economic Theory 70.2 (1996): 312-341.
4. Oyer, Paul. "Why do firms use incentives that have no incentive effects?." The Journal of Finance 59.4 (2004): 1619-1650.
5. Rahman, David. "But who will monitor the monitor?." American Economic Review 102.6 (2012): 2767-97.

Overview

Introduction

- Generally, tournaments ignore the information in an inefficient way.

This paper:

- Studies the environment under which tournaments and independent contracts perform better than the other.
- In its setup: one risk-neutral principal employs many risk-averse agents.

Notation and Assumptions

- $N = \{1, 2, ...n\}$ risk averse agents, each with utility:

$$U^i(m_i, x_i) = u(m_i) - x_i,$$

  where m_i, and x_i are agent's income and effort, respectively, and $u(.)$ is strictly increasing and strictly concave.
- The output of agent i, y_i, depends stochastically on his effort level x_i:

$$y_i = z_i + \eta,$$

  where z_i is a random variable whose distribution depends on x_i, and η is a random variable affecting all of the agents.
- Let $F(\ ; x_i)$ denote the conditional CDF for z_i given x_i; since the agents are identical ex ante, F does not depend on i.

Notation and Assumptions - Continued

- The agents observe private signals $\sigma_i \in \mathbb{R}$ about η before choosing their effort level. Let G denote the CDF for (η, σ).
- Assume that z_i and (η, σ) are independent and η has zero mean.

The principal:

- Only observes the outputs, $y = (y_1, ..., y_n)$, and not the agents' effort levels.
- Pays $v(y) \equiv u(R(y))$ in *util* for a reward function $R(y)$.
- Wants to maximize:

$$\mathbb{E}[y - u^{-1}(v(y))],$$

subject to agents' ICs and IRs.

Independent Contracts

- The problem of the principal is to choose the optimal (v, X) subject to agents' constraints:

$$S_{ci}(G) \equiv \{(v, X) | v : \mathbb{R}_+ \to [0, B],\ X : \mathbb{R} \to \mathbb{R}_+;$$

$$X(\sigma_i) \in \arg\max_x \int v(y) \int f(y - \eta; x) dG(\eta, \sigma_{-i} | \sigma_i) dy - x \quad \forall \sigma_i,$$

$$\int \int (v(y) - X(\sigma_i))\ f(y - \eta; X(\sigma_i)) dG(\eta, \sigma) dy \geq u^0\ \}$$

- The expected payoff of the principal is:

$$P_{ci}(v, X, G) \equiv \int \int (y - u^{-1}(v(y)))\ f(y - \eta; X(\sigma_i)) dy\ dG(\eta, \sigma)$$

Tournaments

- The prizes are $w = (w_1, w_2, ..., w_n)$ in *utils*.
- In tournaments, the rank order of the outputs depend only on the z_is and NOT on η:

$$y_i \geq y_j \Longleftrightarrow z_i \geq z_j.$$

- The problem of the principal is to choose the optimal $(w, \bar{x})$ subject to agents' constraints:

$$S_T(n) \equiv \{(w, \bar{x}) | w \in [0, B]^n, \bar{x} \in \mathbb{R}_+;$$

$$\bar{x} \in \arg\max_x \frac{1}{n} \Sigma_{j=1}^n w_j \int \frac{f(z; x)}{f(z; \bar{x})} \phi_{jn}(z; \bar{x}) dz - x,$$

$$\frac{1}{n} \Sigma_{j=1}^n w_j - \bar{x} \geq u^0 \}$$

where ϕ_{nj} is the j^{th} order statistic of $(z_1, ..., z_n)$.

- The expected payoff of the principal is:

$$P_T(n, w, \bar{x}) \equiv \int \int y\ f(y - \eta; \bar{x}) dG(\eta, \sigma)\ dy - \frac{1}{n} \Sigma_{j=1}^n u^{-1}(w_j)$$

Main Results

- A1: There is no common error term. i.e. if:

$$\int_{\sigma\in\mathbb{R}^n} dG(\eta,\sigma) \equiv \begin{cases} 0 & \textit{for} \quad \eta < 0 \\ 1 & \textit{for} \quad \eta \geq 0 \end{cases}$$

* *Proposition 1*: For any F, G satisfying A1, and $n \geq 2$, given $(w, \bar{x}) \in S_T(n)$, there exists $(v, X) \in S_{ci}(G)$, $i = 1, ..., n$, such that:

$$P_{ci}(v, X, G) \geq P_T(n, w, \bar{x}), \quad i = 1, ..., n.$$

The inequality is strict unless $(w, \bar{x}) = ((u^0, u^0, ..., u^0), 0)$.

- *Corollary 1*: Let F, G satisfying A1, and $n \geq 2$ be given. Then:

$$\max_{(v,X)\in S_{ci}(G)} P_{ci}(v, X, G) \; \geq \max_{(w,\bar{x})\in S_T(n)} P_T(n, w, \bar{x}), \; \forall i.$$

Main Results - Continued

- A2: each member of the sequence $\{G_k\}_{k=1}^{\infty}$ has a density function g_k such that:

$$\int g_k(\eta, \sigma_{-i}|\sigma_i)d\sigma_i \equiv g_{ki}(\eta|\sigma_i) < \frac{1}{k}, \quad \forall \eta, \sigma_i, i.$$

* *Proposition 2*: Let F, $\{G_k\}_{k=1}^{\infty}$ satisfying A2, and $n \geq 2$, be given. Assume that $f_x(z; x)$ is a function of bounded variation in z, for all $x \geq 0$, and that the bound, M, is uniform in x. Then, there exists K such that for all $k > K$:

$$\max_{(w,\bar{x}) \in S_T(n)} P_T(n, w, \bar{x}) \ \geq \max_{(v,X) \in S_{ci}(G_k)} P_{ci}(v, X, G_k), \ \forall i.$$

The inequality is strict unless the lefthand side is equal to P^0.

Conclusion

Tournaments:

1. Tend to reduce the randomness of agent's compensation by filtering out the common shock.
2. Also tend to increase the randomness in agent's compensation by making his reward depend on the idiosyncratic shocks of his peers.

- Propositions 1, and 2, show that the relative (dis)advantage of tournaments versus contracts depends on which effect dominates.

The reason why tournaments are popular is that:

- It is much easier for the principal to determine the agents' rankings rather than to measure their effort levels.
- Tournaments filter out common shocks which is important if the distribution is unknown (such as nonstationary environments).

Implications for Other Literatures

1. Rosen, Sherwin. "Prizes and incentives in elimination tournaments." (1985).
2. Ehrenberg, Ronald G., and Michael L. Bognanno. "Do tournaments have incentive effects?." Journal of political Economy 98.6 (1990): 1307-1324.
3. Dixit, Avinash. "Strategic behavior in contests." The American Economic Review (1987): 891-898.
4. Nalebuff, Barry J., and Joseph E. Stiglitz. "Prizes and incentives: towards a general theory of compensation and competition." The Bell Journal of Economics (1983): 21-43.

Thanks for your attention!